Mini-course:

Optimal Control of Space Trajectories using GEKKO

Lecture 1: Introduction

Jhonathan Murcia-Piñeros

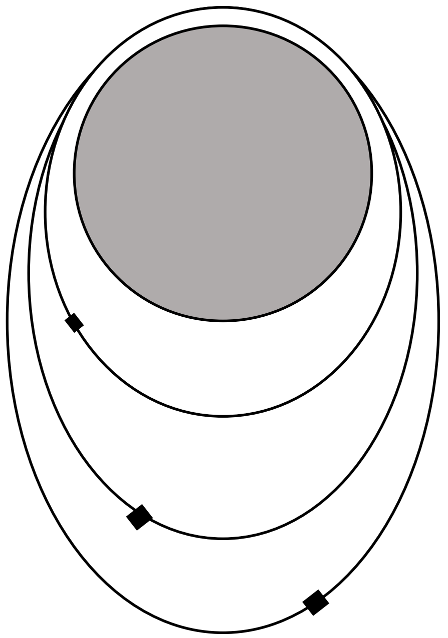
jhonathan.pineros@unifesp.br

CBDO - 2024

04/12/2024

Summary

I. Motivation.
II. Formulating the Optimal Control Problem.
III. Example for Verification and validation.
IV. Application in aerogravity assisted maneuvers
V. GEKKO.



I. Motivation and goal

- The use of software to solve optimal control problems (OCP) is a highly demanded skill for professionals in the fields of Astrodynamics, Guidance, Navigation, and Control.
- The leading software used in this field is commercial and expensive, including the add-on to apply in OCP.
- GEKKO package in Python is an alternative.
- Last year, GEKKO was used to analyze aero maneuvers above Earth, Venus, and Mars. This was part of the results of a postdoctoral Internship supported by the FAPESP.
- In this mini-course are presented two examples of the implementation of GEKKO to solve optimal control problems in space trajectories.
- To this aim, the OCP is formulated and solved via non-linear programming (NLP).

Papers

- Along this course are discussed the formulations presented in the papers:

  - Murcia-Piñeros, J., Prado, A. F., Dimino, I., & de Moraes, R. V. (2024). Optimal gliding trajectories for descent on Mars. Applied Sciences, 14(17), 7786. Doi: 10.3390/app14177786

  - Piñeros, J. O. M., Bevilacqua, R., Prado, A. F., & de Moraes, R. V. (2024). Optimizing aerogravity-assisted maneuvers at high atmospheric altitude above Venus, Earth, and Mars to control heliocentric orbits. Acta Astronautica, 215, 333-347. Doi: 10.1016/j.actaastro.2023.12.017

  - Murcia-Piñeros, J., Bevilacqua, R., Gaglio, E., Prado, A. B., & De Moraes, R. V. (2024). Optimization of Aero-gravity assisted maneuvers for spaceplanes at high atmospheric flight on Earth. In AIAA SCITECH 2024 Forum (p. 1459). Doi: 10.2514/6.2024-1459

II. The Optimal Control Problem

A nonlinear continuous-time, time-invariant system $\dot{\boldsymbol{X}}(t)$ of Ordinary Differential Equations (ODE) is described in Eq. (1). The system $\dot{\boldsymbol{X}}(t)$ is composed of the kinematics and dynamics equations of the system, which are a function of the state vector ($\boldsymbol{X}(t)$), and the control vector ($\boldsymbol{U}(t)$) [1, 2]:

$$\dot{\boldsymbol{X}} = f(\boldsymbol{X}(t), \boldsymbol{U}(t), t) \quad (1)$$

The system is changing subject to the algebraic path constraints:

$$\boldsymbol{X}_I \leq \boldsymbol{X}(t) \leq \boldsymbol{X}_U \quad (2)$$

$$\boldsymbol{U}_I \leq \boldsymbol{U}(t) \leq \boldsymbol{U}_U \quad (3)$$

And the end point conditions:

$$\boldsymbol{e}\left(\boldsymbol{X}(t_0), t_0, \boldsymbol{X}\left(t_f\right), t_f\right) = 0 \quad (4)$$

The system is composed by the continuous-time $t \in \mathbb{R}_{\geq 0}$, the state vector $\boldsymbol{X}(t) \in \mathbb{R}^n$, the initial conditions are $\boldsymbol{X}(0) = x_0$; $x_0 \in \mathbb{R}^n$. $\boldsymbol{U}(t) \in \mathbb{R}^m$ is the control input subject to $\boldsymbol{U}(t) \in \mathbb{U} \subset \mathbb{R}^m$.

Formulation of the Optimal Control Problem

$$\mathbf{J}\left(\boldsymbol{X}(\cdot),\boldsymbol{U}(\cdot),t_0,t_f\right)=\mathbf{M}\left(\boldsymbol{X}\left(t_f\right),t_f\right)v+\int_{t_0}^{t_f}\mathbf{L}(\boldsymbol{X}(t),\boldsymbol{U}(t),t)\,dt \qquad (5)$$

Maximize or minimize: $\max\limits_{\mathbf{U}(t),t_f}\mathbf{J}\left(t_f\right)$

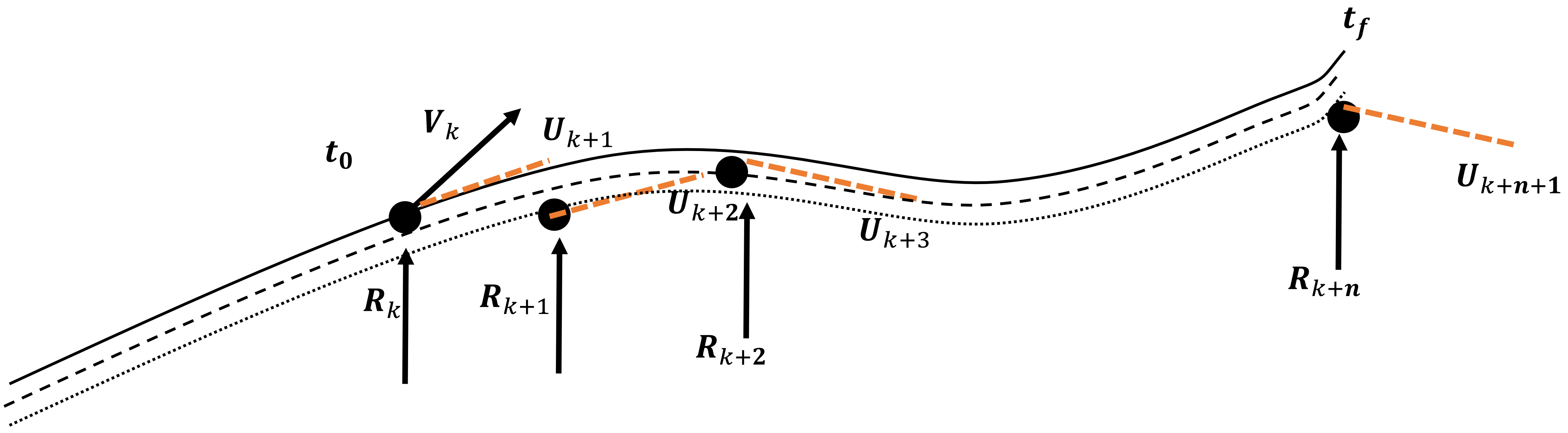
For the objective functionals:

Cost 1; **Cost 2**; **Cost n** (6)

Subject to: $\forall t \in [t_0, t_f\ ]$,

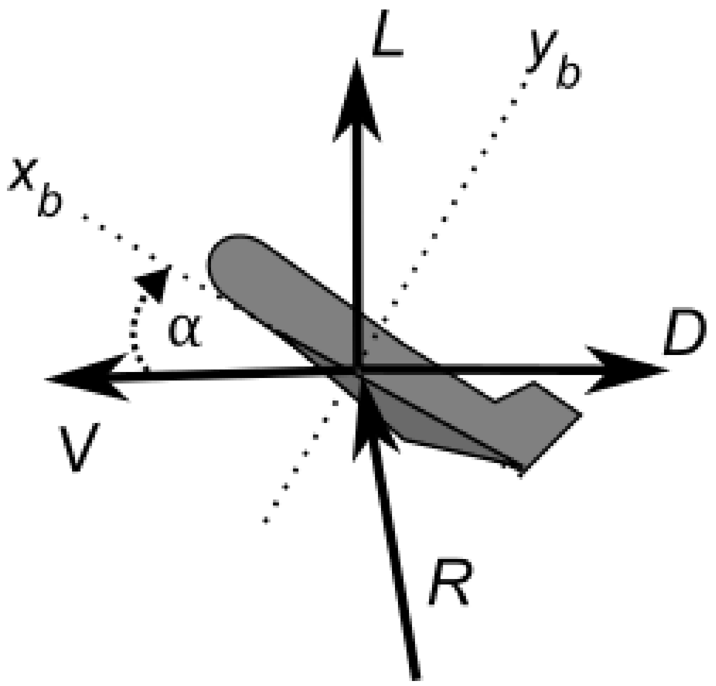
The Dynamic and constraints: Eqs. (1) – (4)

To find the control vector: $\mathbf{U}(t)$



8

III. Example – space shuttle gliding reentry



$$\dot{R} = V \sin\gamma \quad (7)$$

$$\dot{\theta} = \frac{V\cos\gamma\cos A}{R\cos\varphi} \quad (8)$$

$$\dot{\varphi} = \frac{V\cos\gamma\sin A}{R} \quad (9)$$

$$\dot{V} = \frac{-D(\alpha)}{m} - g\sin\gamma \quad (10)$$

$$\dot{\gamma} = \frac{L(\alpha)\cos\beta}{mV} - \frac{g\cos\gamma}{V} + \frac{V\cos\gamma}{R} \quad (11)$$

$$\dot{A} = \frac{L(\alpha)\sin\beta}{mV\cos\gamma} - \frac{V\tan\varphi\cos A\cos\gamma}{R} \quad (12)$$

$$\boldsymbol{U}\{\alpha(t), \beta(t)\} \quad (13)$$

- **One way to obtain the optimal trajectory is solving the OCP via nonlinear programming (NLP).**

- **Two software are used to verify and validate the results.**

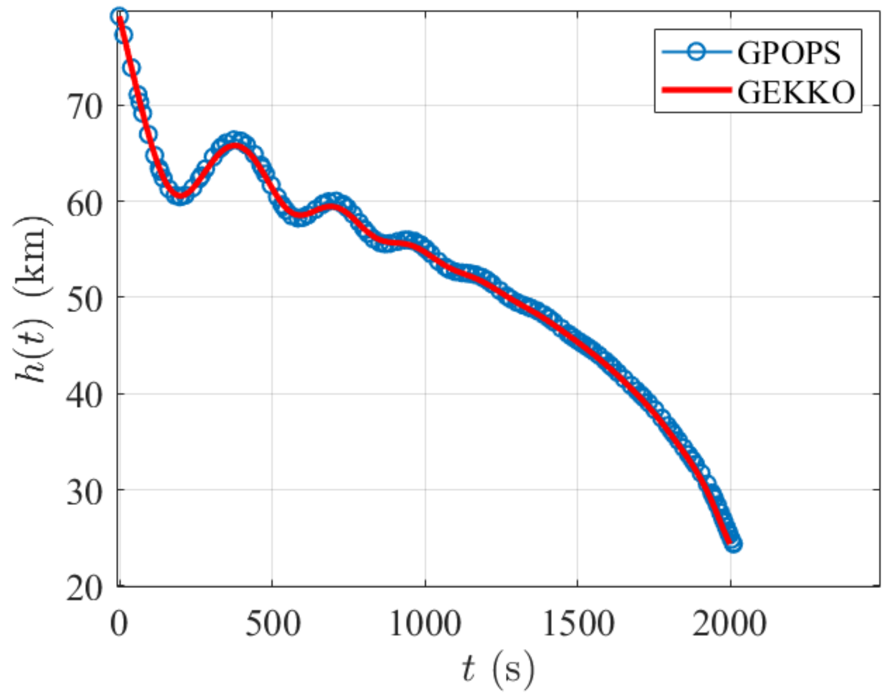
Verification and validation, V&V
The reentry problem [2, 3].

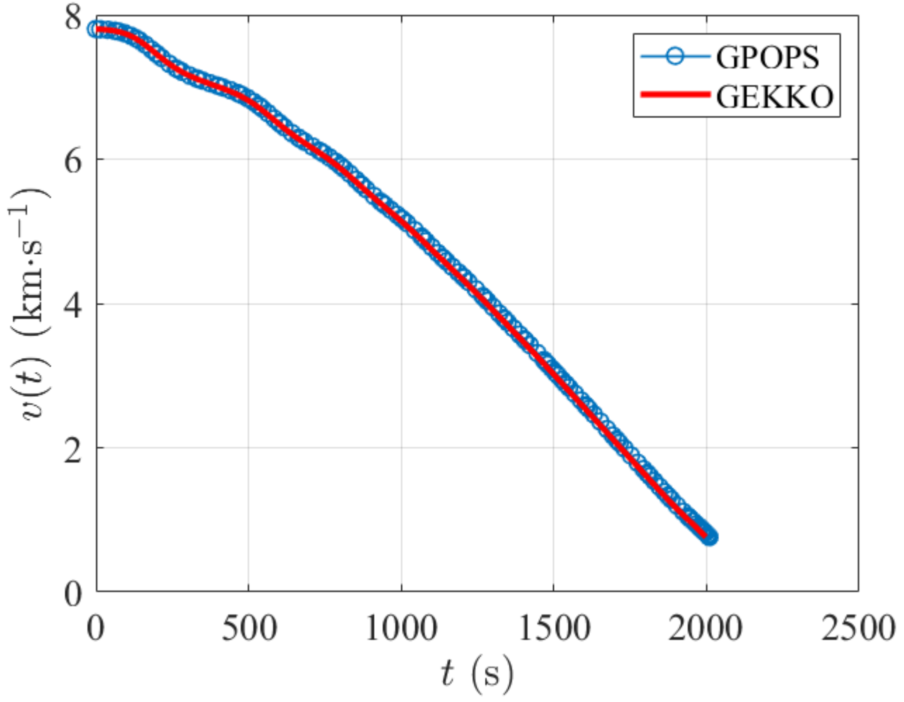
- Initial conditions:
  - Altitude: 79248 m
  - Velocity: 7802.88 m/s
  - FPA: -1°

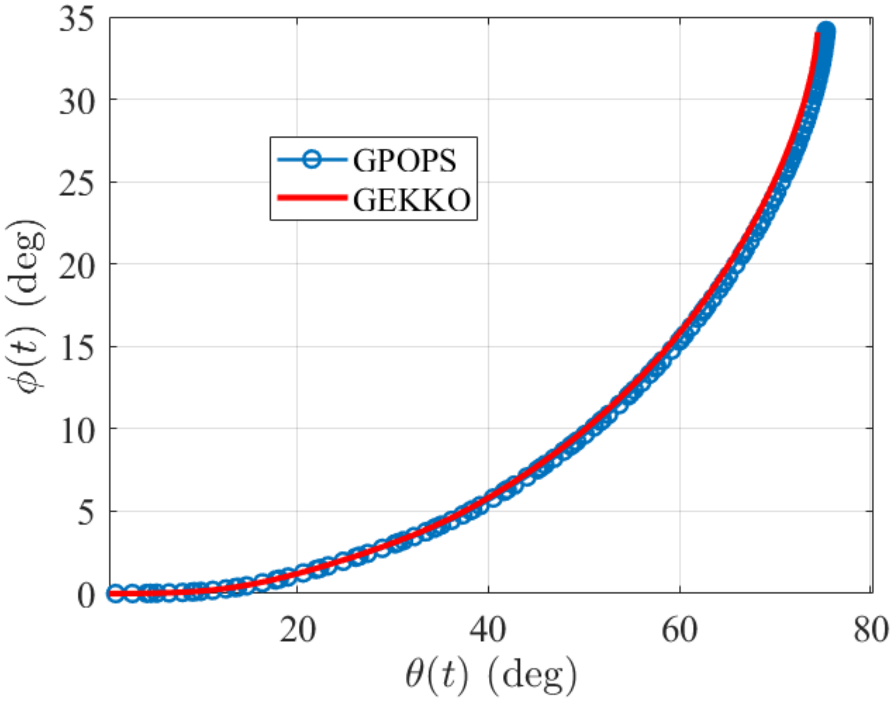
- Final conditions:
  - Altitude: 24384 m
  - Velocity: 762 m/s
  - FPA: -5°

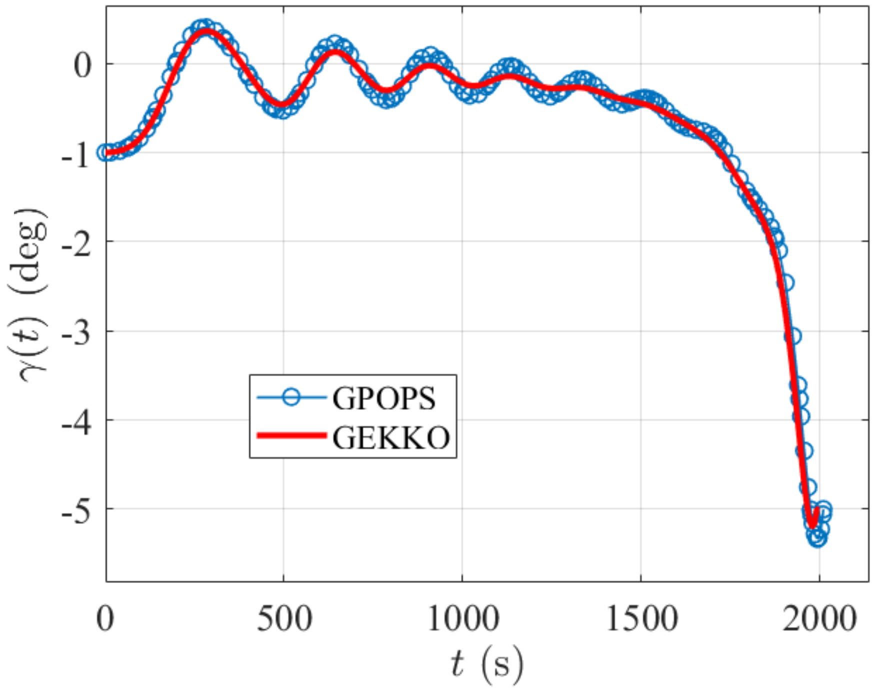
Objective: Maximize cross-range or:

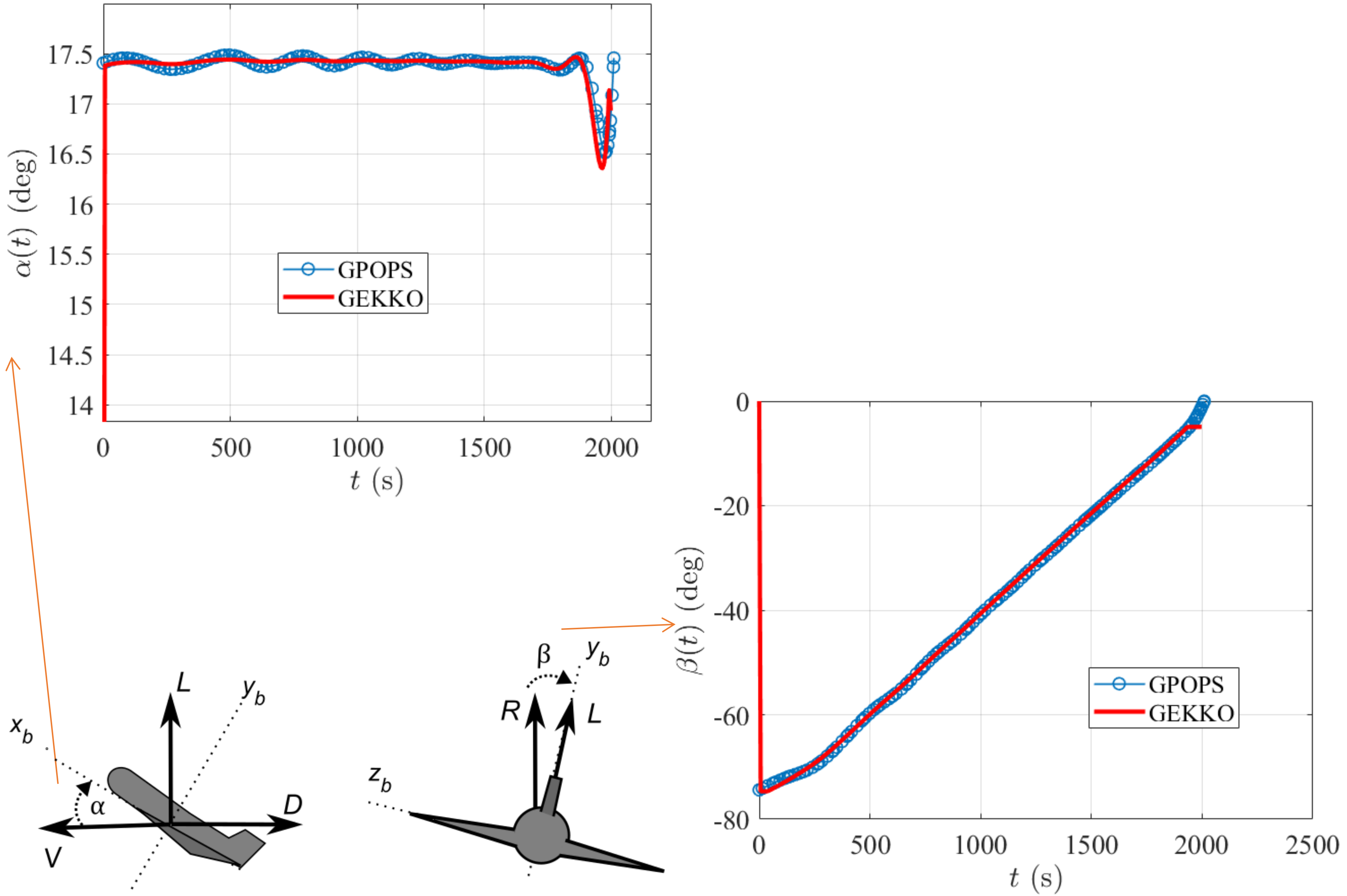
$$\boldsymbol{J} = \max_{t_f}(\varphi)$$


GPOPS
GEKKO
70
60
50
40
30
20
$h(t)$ (km)
0
500
1000
1500
2000
t (s)


GPOPS
GEKKO
8
6
4
2
0
$v(t)$ (km·s^{-1})
0
500
1000
1500
2000
2500
t (s)

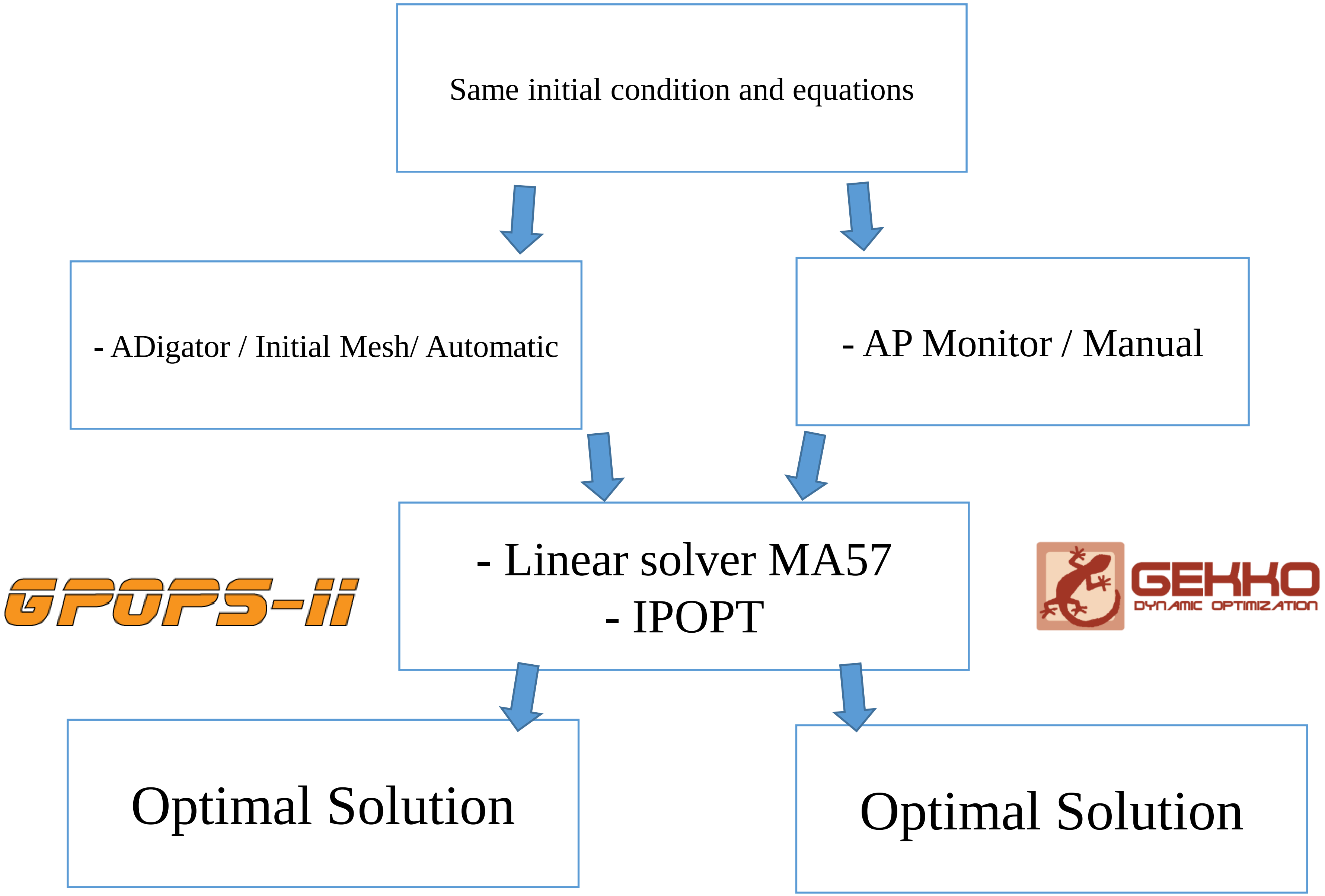

35
30
25
20
15
10
5
0
20
40
60
80
$\phi(t)$ (deg)
$\theta(t)$ (deg)
GPOPS
GEKKO


0
-1
-2
-3
-4
-5
0
500
1000
1500
2000
$\gamma(t)$ (deg)
t (s)
GPOPS
GEKKO



17.5
17
16.5
16
15.5
15
14.5
14
$\alpha(t)$ (deg)
0
500
1000
1500
2000
t (s)
GPOPS
GEKKO
0
-20
-40
-60
-80
$\beta(t)$ (deg)
0
500
1000
1500
2000
2500
t (s)
GPOPS
GEKKO
L
y_b
x_b
α
D
V
β
y_b
R
L
z_b

	GPOPS	GEKKO
Optimal value (rad)	0.59627638896329227	0.594499823031365
Overall NLP error	7.7632331161873700e-9	1.0000000000001961e-11
Obj. function eval.	13	149
Iterations	12	88
IPOPT CPU TIME (S)	0.016	5.909



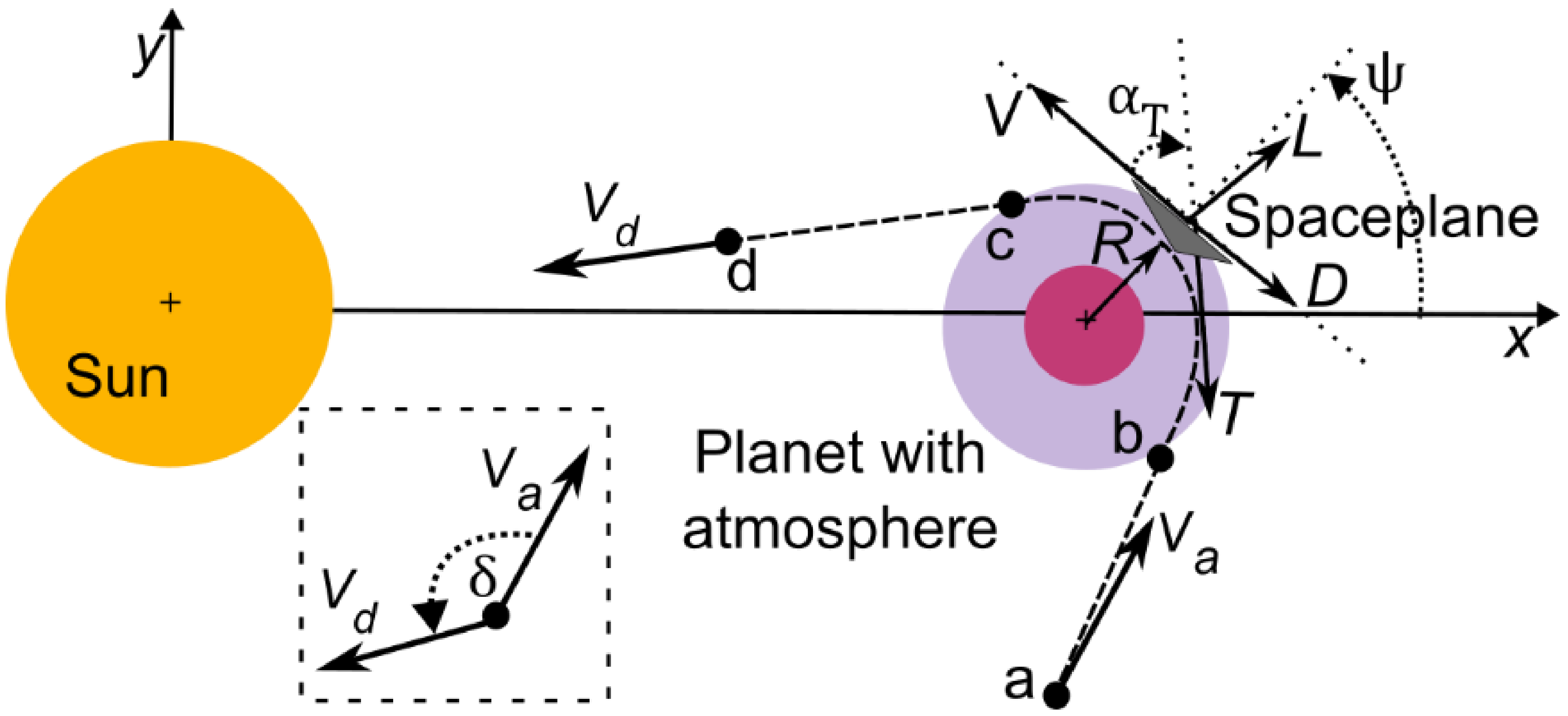
Same initial condition and equations
- ADigator / Initial Mesh/ Automatic
- AP Monitor / Manual
GPOPS-ii
- Linear solver MA57
- IPOPT
GEKKO
DYNAMIC OPTIMIZATION
Optimal Solution
Optimal Solution

IV. More applications: Aero-Gravity Assist Maneuver (AGAM).

- Interaction with the atmosphere of a celestial body during a close approach, exploiting the lift as the dominant aerodynamic force [4-6].

- These maneuvers reduce the cost of the mission during interplanetary flight since they are an alternative to obtaining natural impulses or Delta-V from the environment.

- In other words, **the optimization of AGAM helps to save fuel**.



The geometry of the powered aero-gravity assist maneuver (PAGAM).

The Optimal Control Problem - Constraints

$$t_0 < t \leq t_f$$

$$30\ s \leq t_f \leq 600\ s$$

$$75\ km \leq h(t) \leq 90\ km$$

$$V_{escape} \leq V(t_f)$$

$$-180\ deg \leq \theta(t) \leq 180\ deg$$

$$-90\ deg \leq \varphi(t) \leq 90\ deg$$

$$-10\ deg \leq \gamma(t) \leq 10\ deg$$

$$0\ deg \leq A(t) \leq 359.9\ deg$$

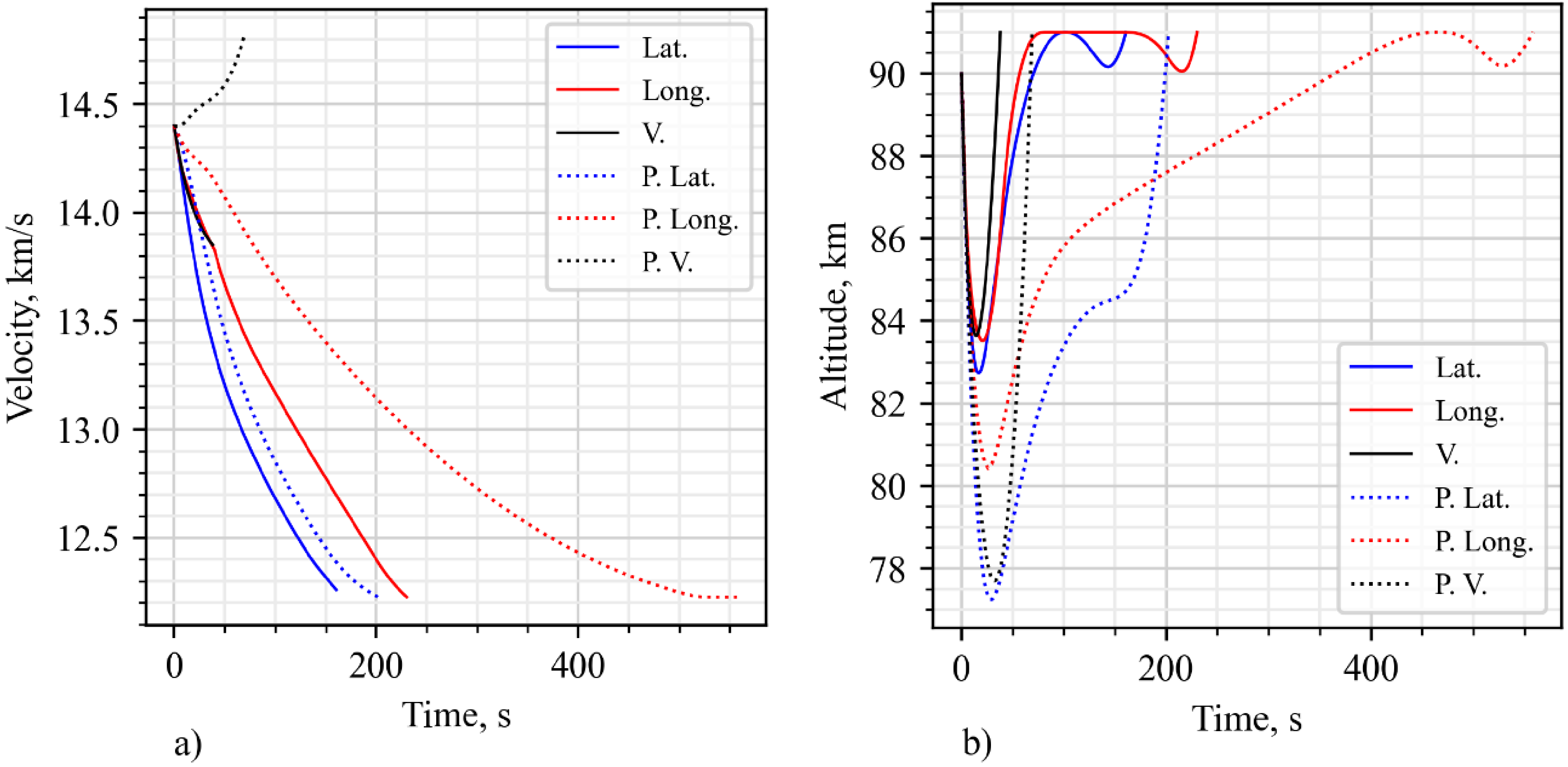
$$m_{min} \leq m(t) \leq m_{max} \qquad (10)$$

$$-4\ deg \leq \alpha(t) \leq 28\ deg$$

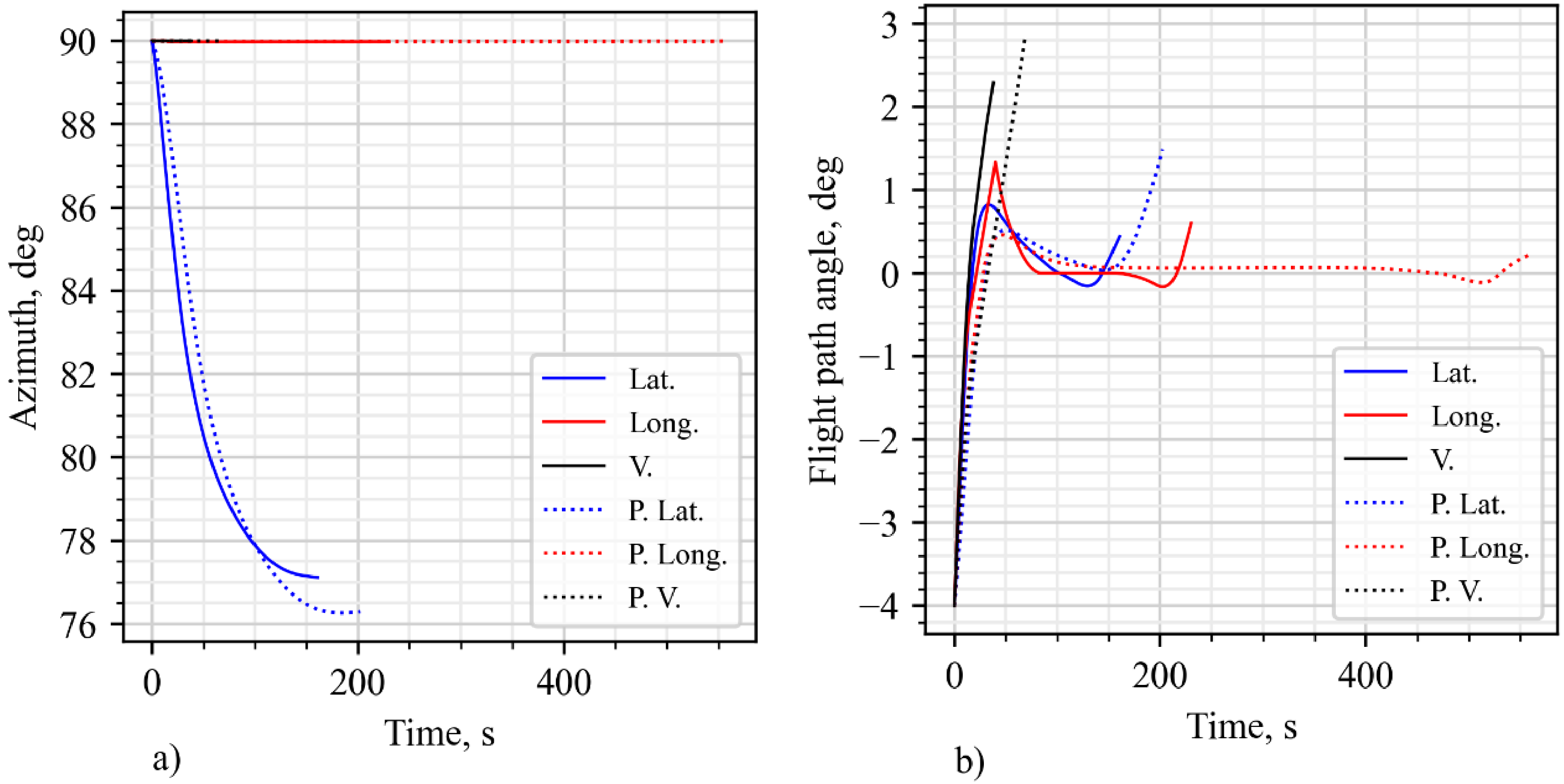
$$-180\ deg \leq \beta(t) \leq 180\ deg$$

$$0\ N \leq T(t) \leq 266\ kN$$

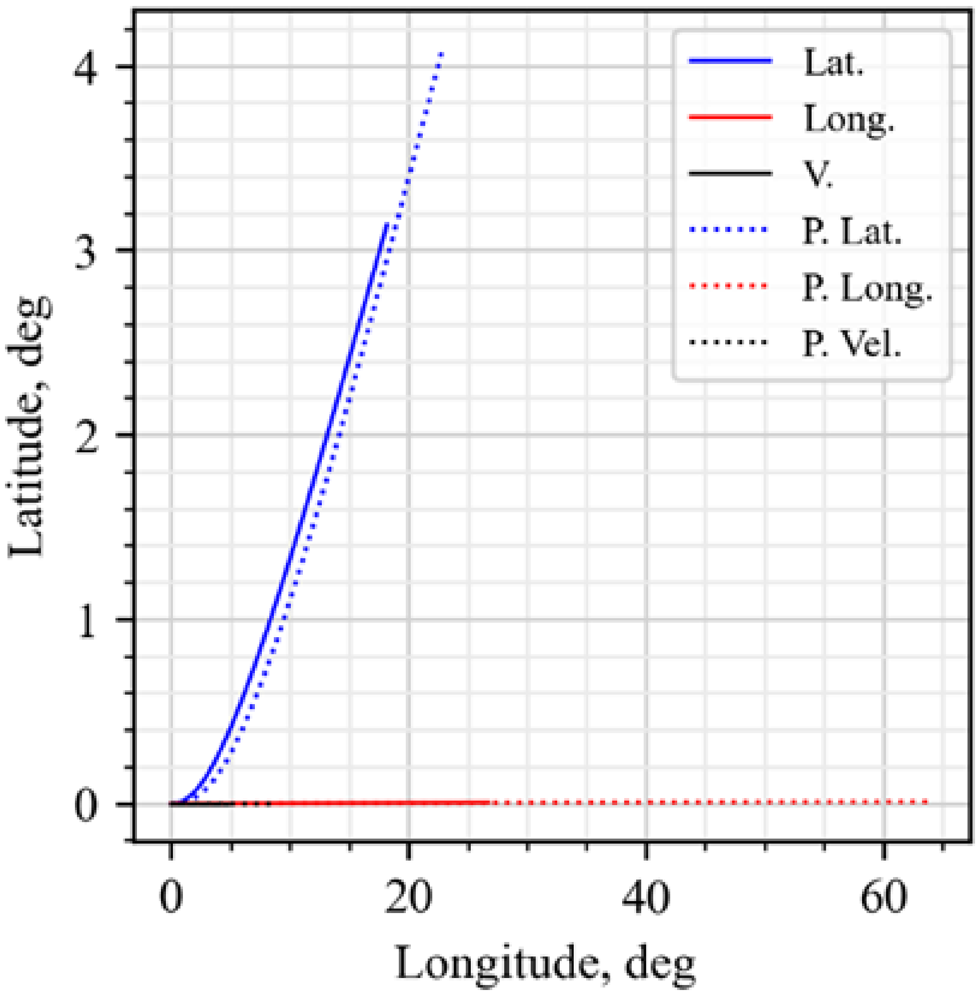
Results and discussion –AGAM and PAGAM.



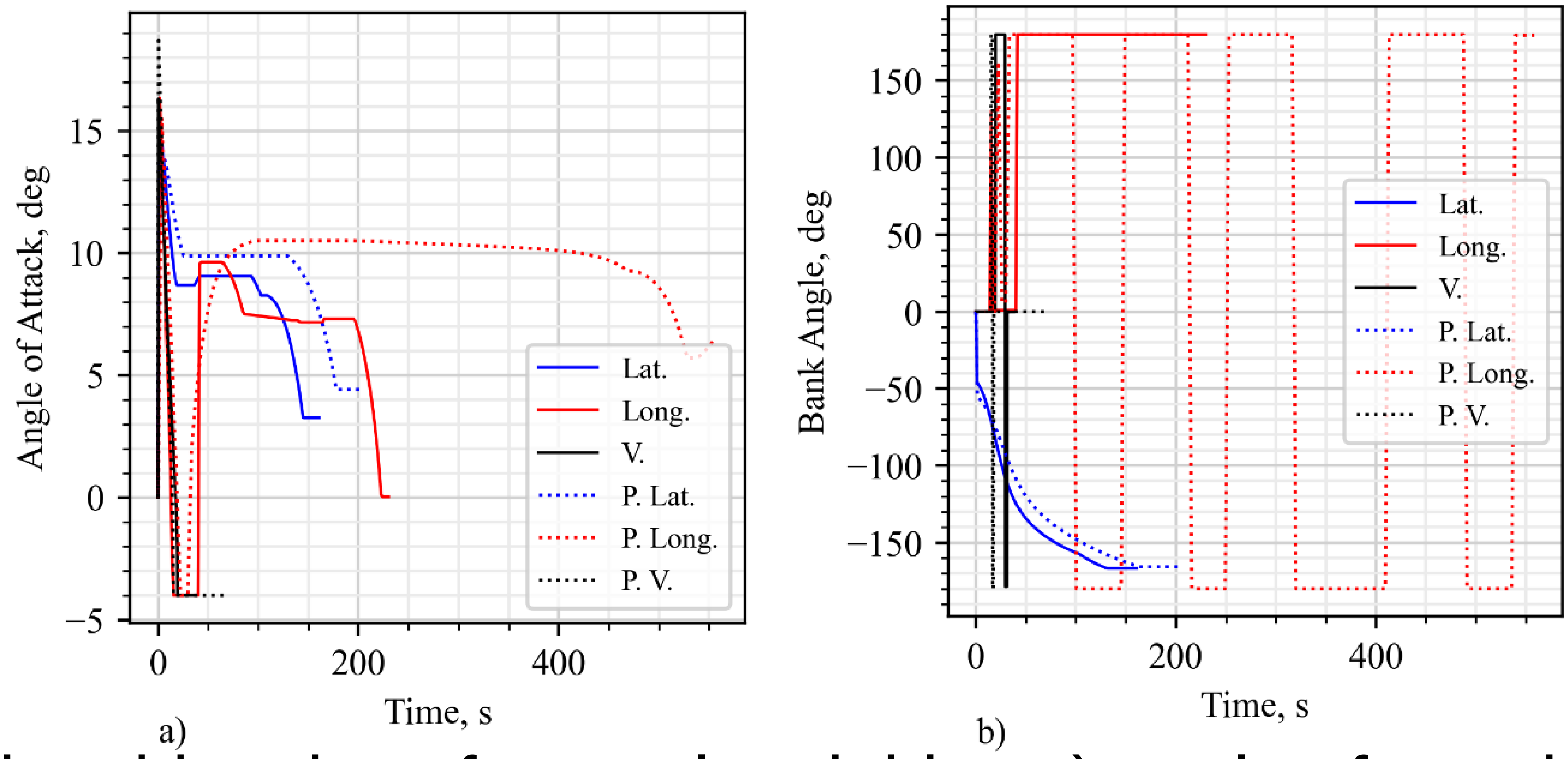
Time histories a) velocity and b) altitude.



Time histories a) azimuth and b) flight path angle.



Longitude vs latitude.

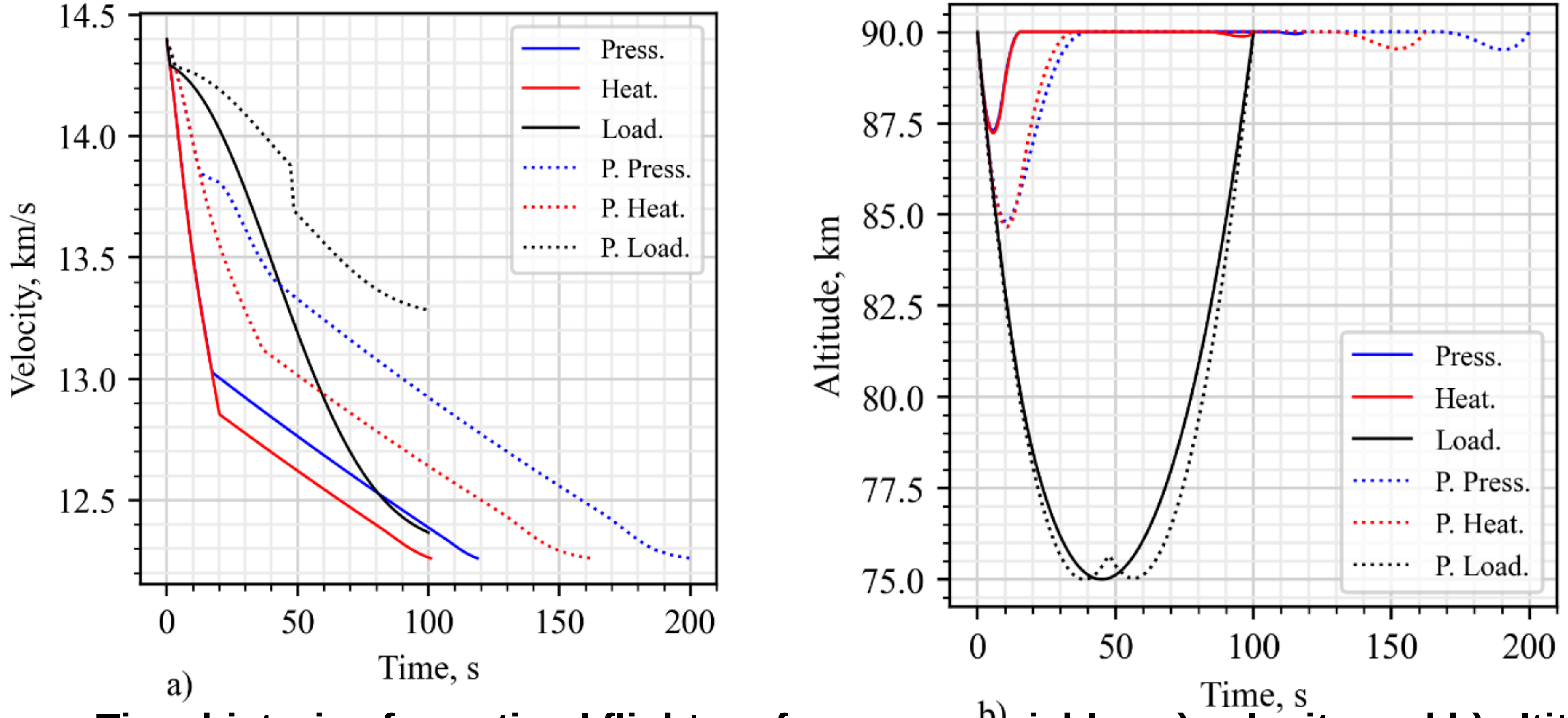


Time histories of control variables a) angle of attack and b) bank angle.

- **Cost 4:** $\mathbf{J}\left(t_f\right) = \int_{t_0}^{t_f} \dot{q}_c \, . dt$

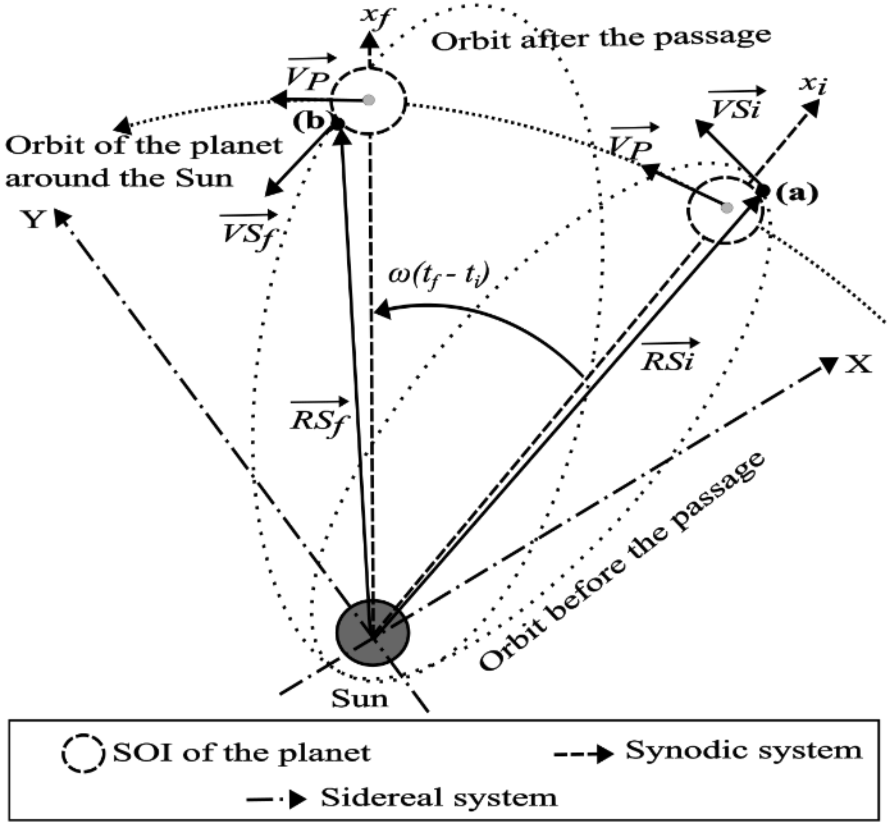
- **Cost 5:** $\mathbf{J}\left(t_f\right) = 0.5 \int_{t_0}^{t_f} \rho V^2 \, . dt;$ (14)

- **Cost 6:** $\mathbf{J}\left(t_f\right) = \int_{t_0}^{t_f} n \, . dt$

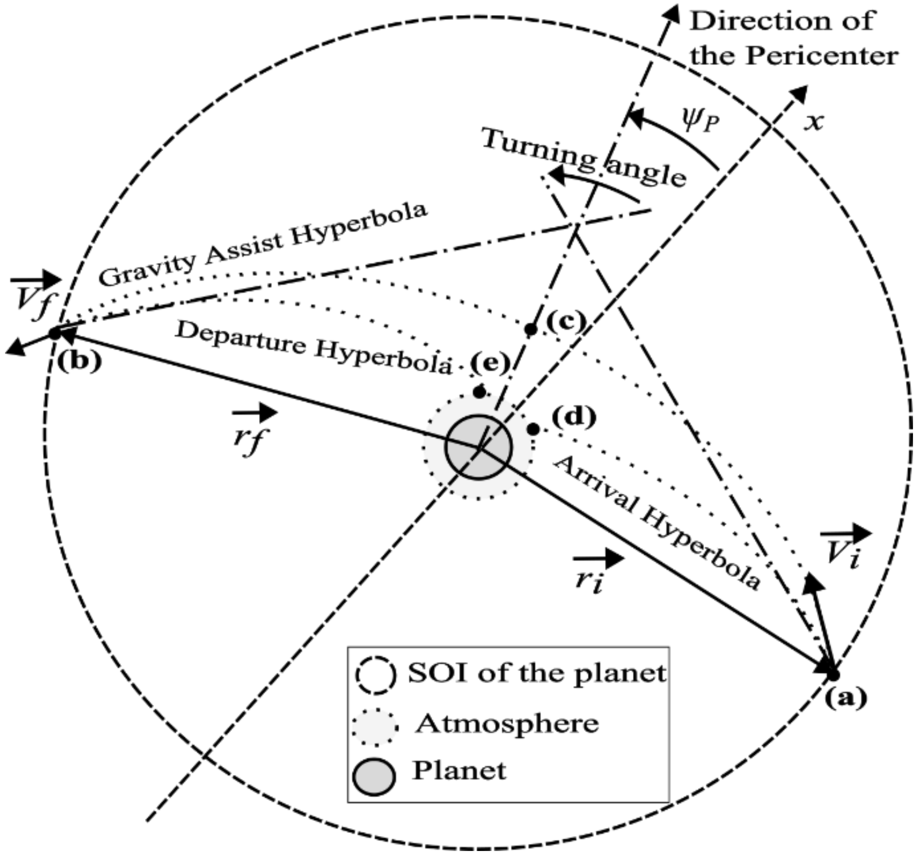


Time histories for optimal flight performance variables a) velocity and b) altitude.

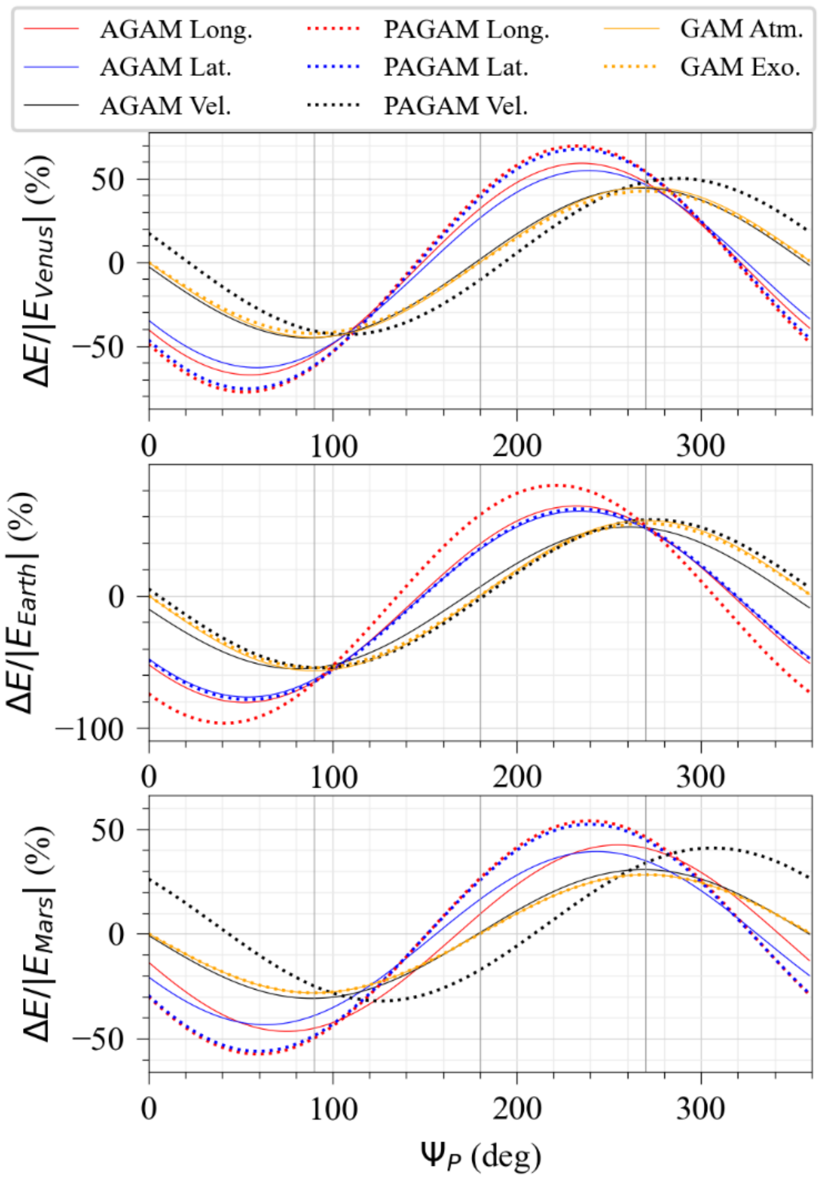
Effects of the maneuvers around the Sun



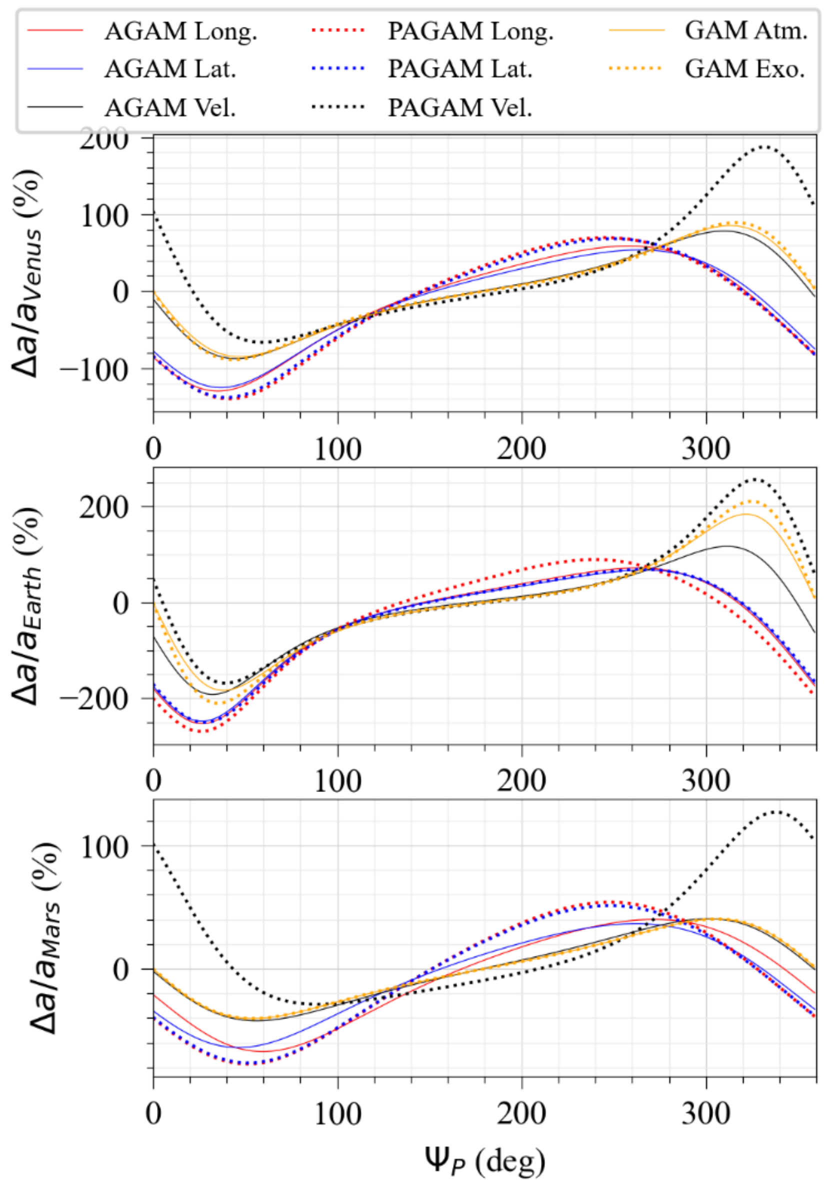
Geometry of the maneuver around the Sun.



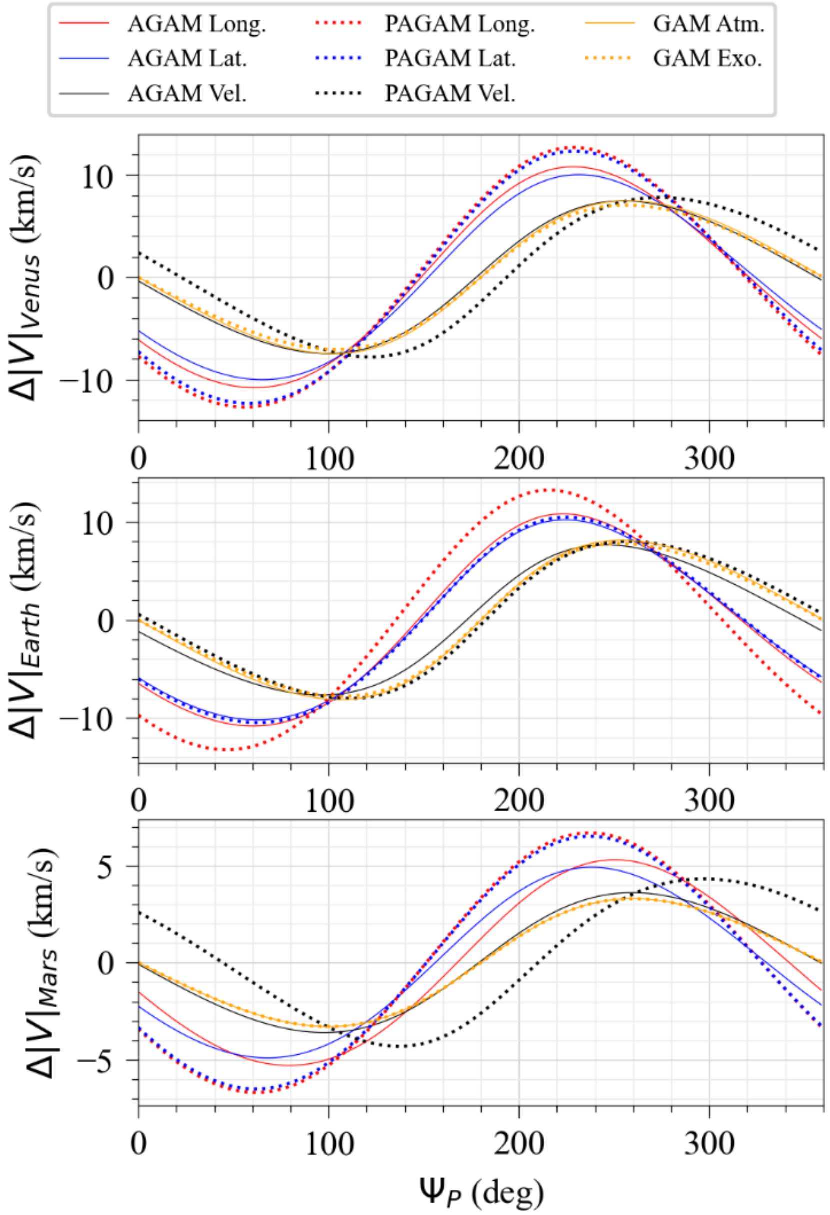
Geometry of the maneuver around the planet.



Heliocentric energy changes.



Variation of heliocentric semimajor axis.



Variation of heliocentric velocity.

V. GEKKO [7, 8]

- GEKKO is a Python package or library used for optimization and machine learning.

- Some applications include: solving linear and nonlinear equations, interpolation, regression, dynamic simulation, real-time optimization,

Installation

- Requirements: Python, code editor such as VS Code, Jupyter Notebook, Spyder.
- Installation, in terminal: `pip install gekko`

More info

- https://gekko.readthedocs.io/en/latest/#
- https://apmonitor.com/wiki/index.php/Main/GekkoPythonOptimization
- www.youtube.com/@apm

References

[1] Armellin, R., Lavagna, M., and Ercoli-Finzi, A.,(2006) Aero-gravity assist maneuvers: controlled dynamics modeling and optimization, Celestial Mechanics and Dynamical Astronomy, Vol. 95,, pp. 391–405. doi: 10.1007/s10569-006-9024-y

[2] Betts J. (2010) Practical Methods for Optimal Control Using Nonlinear Programming. 3 Ed..

[3] Patterson, M., Rao, A. (2016) GPOPS II Manual. V 2.3. Available at: https://gpops2.com/resources/gpops2UsersGuide.pdf

[4] Murcia-Piñeros, J., Prado, A. F., Dimino, I., & de Moraes, R. V. (2024). Optimal gliding trajectories for descent on Mars. Applied Sciences, 14(17), 7786. Doi: 10.3390/app14177786

[5] Piñeros, J. O. M., Bevilacqua, R., Prado, A. F., & de Moraes, R. V. (2024). Optimizing aerogravity-assisted maneuvers at high atmospheric altitude above Venus, Earth, and Mars to control heliocentric orbits. Acta Astronautica, 215, 333-347. Doi: 10.1016/j.actaastro.2023.12.017

[6] Murcia-Piñeros, J., Bevilacqua, R., Gaglio, E., Prado, A. B., & De Moraes, R. V. (2024). Optimization of Aero-gravity assisted maneuvers for spaceplanes at high atmospheric flight on Earth. In AIAA SCITECH 2024 Forum (p. 1459). Doi: 10.2514/6.2024-1459.

[7] Beal, L.D.R., Hill, D., Martin, R.A., and Hedengren, J. D., “GEKKO Optimization Suite”, Processes, Vol. 6, No. 8, 2018. doi: 10.3390/pr6080106.

[8] Hedengren, J. D., Asgharzadeh Shishavan, R., Powell, K.M., and Edgar, T.F., “Nonlinear Modeling, Estimation and Predictive Control in APMonitor”, Computers and Chemical Engineering, Vol. 70, 2014, pp. 133–148. doi: 10.1016/j.compchemeng.2014.04.013.

Thank you!

Questions?